MATH5010 Linear Analysis (2020-21): Homework 6. Deadline: 8 Mar 2021

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard. Please refer to the course web for details.

1. Let X and Y be normed spaces. Show that if a and b are the elements in X with $a \neq b$, then there is a bounded linear map T from X to Y such that $Ta \neq Tb$.

2. Let X be the vector space $\mathbb{R}^2$ endowed with $\|\cdot\|_1$-norm, that is $\|(x_1, x_2)\|_1 := |x_1| + |x_2|$. Let Y be the vector space $\mathbb{R}^2$ endowed with $\|\cdot\|_\infty$-norm, that is $\|(y_1, y_2)\|_\infty := \max(|y_1|, |y_2|)$. Define a linear map $f : X \to \mathbb{R}$ by $f(x_1, x_2) := 2x_1 - 3x_2$. Find an element $a = (a_1, a_2) \in Y$ such that $f(x_1, x_2) := a_1 x_1 + a_2 x_2$ for all $(x_1, x_2) \in X$ and $\|a\|_\infty = \|f\|_{X^*}$.

$* * *$ **End** $* * *$